B.Sc. 3rd Semester (Programme) Examination, 2019-20

MATHEMATICS

Course ID : 32110 **Course Code : SPMTH/304/SEC-1**

Course Title: Logic and Sets

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Unless otherwise mentioned, notations and symbols have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) Construct a truth table for the statement formula: $(p \to q) \to (\sim q \to \sim p)$.

(b) If $A = \{1, 2, 3\}$ and $B = \{2, 3, 4, 5\}$, then find $A \Delta B$ where Δ is the symmetric difference between two sets.

(c) A relation ρ is defined on the set $\mathbb{Z}$ by "$a\ \rho\ b$ if and only if $ab > 0$" for $a, b, \in \mathbb{Z}$. Examine if ρ is reflexive.

(d) Find $\bigcap_{n \in \mathbb{N}} I_n$ where $I_n = \left(0, \frac{1}{n}\right), n \in \mathbb{N}$.

(e) If $A = \{1, 2\}, B = \{1, 2, 3\}$, find $(A \times B) \cap (B \times A)$.

(f) Using Venn-diagram prove that $(A - C) \cup (B - C) = (A \cup B) - C$.

(g) Write down the truth table for biconditional proposition.

(h) What is the negation of the proposition "Some people have no scooter".

2. Answer *any four* questions: 5×4=20

(a) (i) Construct a truth table for the statement form: $(p \wedge q) \vee \sim r$

(ii) Give the negation of the following statements:

$p : 2 + 3 > 1$

q : It is cold. 3+2=5

(b) (i) If p and q are propositions, then show that $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$.

(ii) Write negation of the following statement. If I am ill, then I cannot go to University. 3+2=5

(c) (i) The relation ρ defined on the set $\mathbb{Z}$ by "$a\rho b$ if and only if $a|b$". Examine if ρ is partial order relation.

(ii) Find the equivalence classes determined by the equivalence relation ρ on $\mathbb{Z}$ defined by "$a\rho b$ if and only if $a-b$ is divisible by 5" for $a, b \in \mathbb{Z}$. 3+2=5

(d) A, B, C are subsets of a universal set S.

(i) Prove that $(A \cup B) \cap (A \cup B') \cap (A' \cup B) = A \cap B$.

(ii) If $A \cup B = A \cup C$ and $A \cap B = A \cap C$, prove that $B = C$. 3+2=5

(e) A, B, C are subsets of a universal set S. Prove that

(i) $[A \cap (B \cup C)] \cap [A' \cup (B' \cap C')] = \phi$

(ii) $(A - B) \times C = (A \times C) - (B \times C)$ 2+3=5

(f) Each student of a class speaks at least one language of Hindi and Bengali. If 40 speak Bengali, 16 speak Hindi and 8 speak both Hindi and Bengali; find the number of students in the class.

3. Answer *any one* question: 10×1=10

(a) (i) Let ρ be an equivalence relation on a set S and $a, b \in S$. Then $cl\,(a) = cl\,(b)$ if and only if $a\ \rho\ b$.

(ii) Prove that $A \cap (B \Delta C) = (A \cap C)$ when $A. B. C.$ are subsets of a universal set ∪. 5+5=10

(b) (i) Show that the propositions $\sim(p \wedge q)$ and $\sim p \vee \sim q$ are logically equivalent.

(ii) Construct the table for $(a \vee b) \leftrightarrow \big[\big((\sim a) \wedge c\big) \rightarrow (b \wedge c)\big]$.

(iii) Let R and S be the following relations on

$A = \{1, 2, 3\}$

$R = \{(1,1), (1,2), (2,3), (3,1), (3,3)\}$

$S = \{(1,2), (1,3), (2,1), (3,3)\}$

Find $R \circ S,\ R^c$ 3+4+(2+1)=10

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